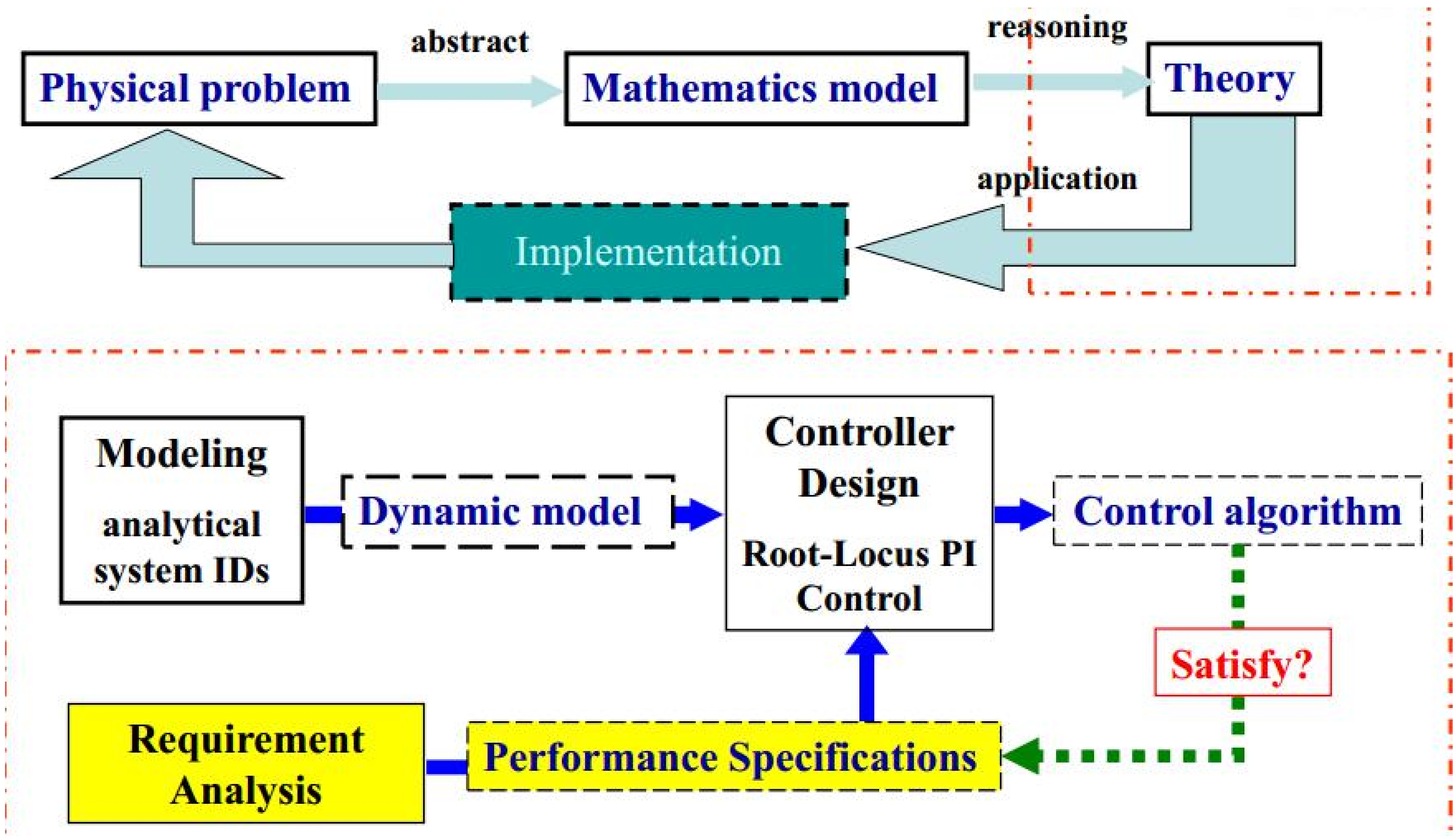
CHAPTER 2

Time Response of Linear System


Physical problem
abstract
Mathematics model
reasoning
Theory
application
Implementation
Modeling
analytical
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Dynamic model
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Control
Control algorithm
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Outline

2.1 Transient response and steady-state response

2.2 Typical test signal

2.3 System specifications in time-domain

2.4 System evaluation criteria

Basic requirements for control systems

- The system is expected to be affected Only by control input, NOT by disturbance.

- Two-fold tasks:

  (1) how the system responses to the external input?

  (2) how the system resists disturbances?

Basic requirements for control systems

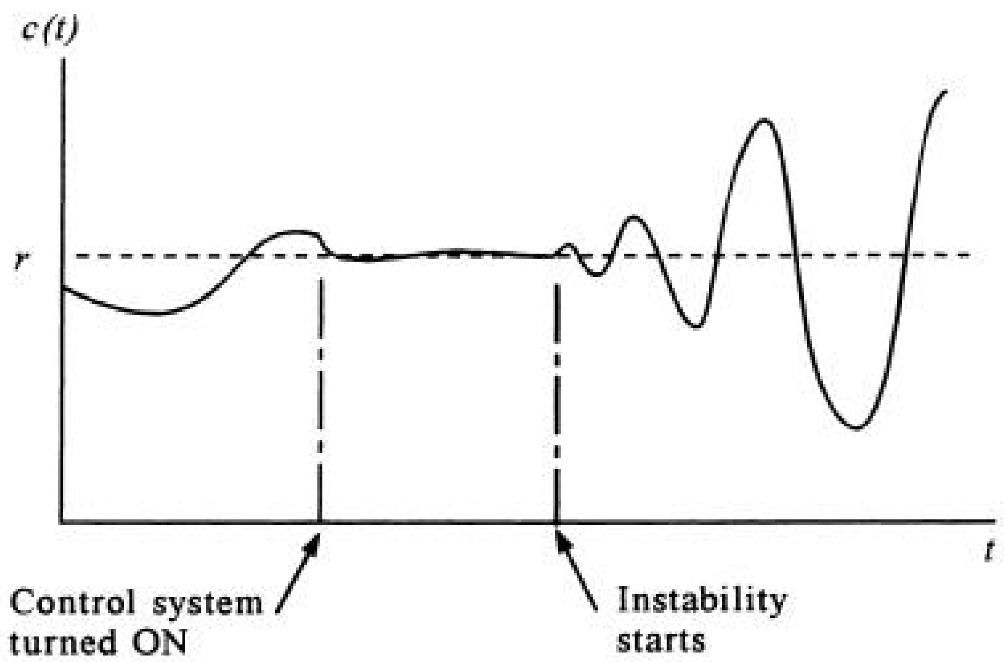
A suitable control system should have some of the following properties:

1. Stability
2. Accuracy
3. Swiftness

Basic requirements for control systems

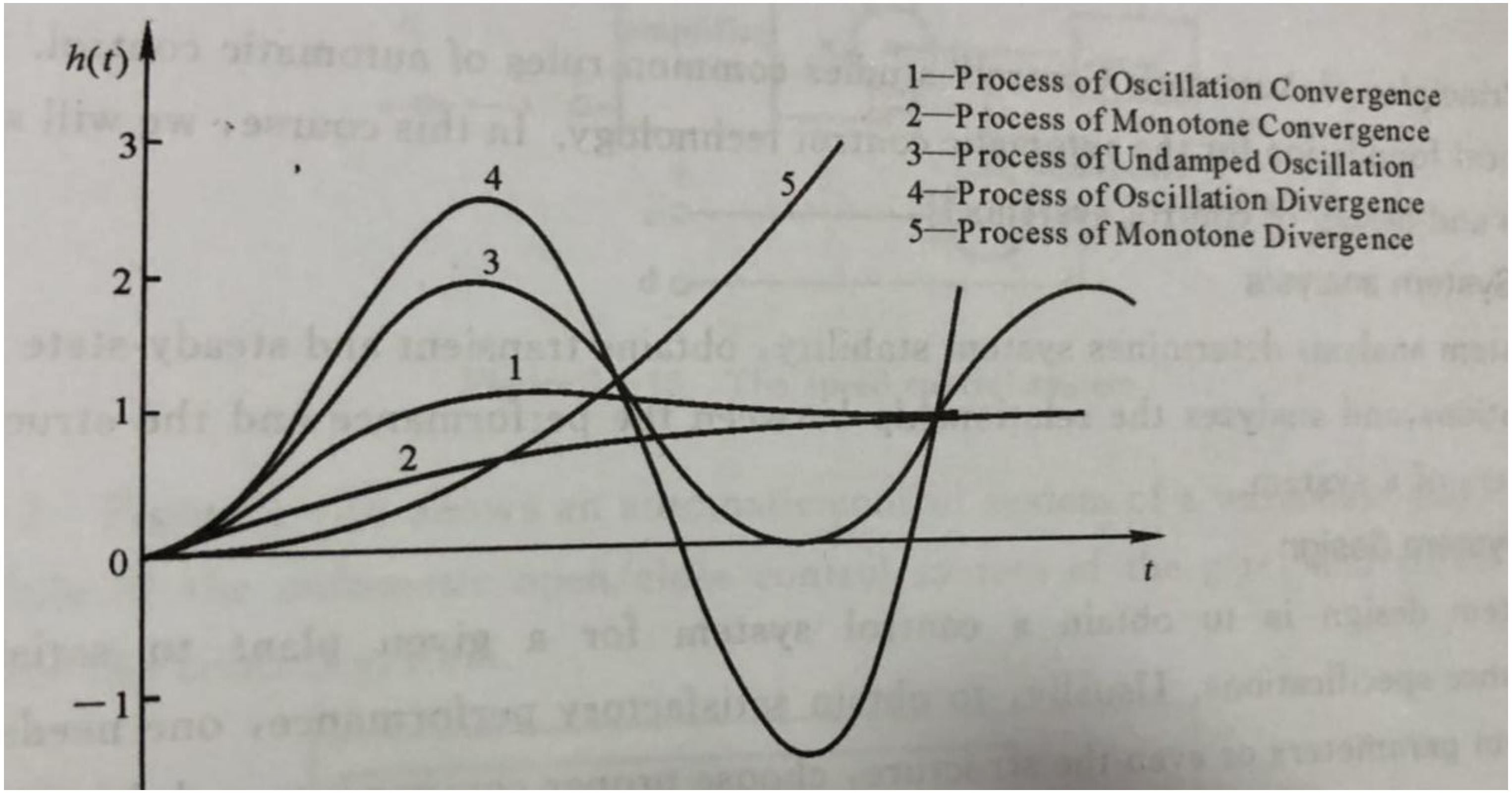
Definitions

1. Stability (**Basic requirement**): refers to ability of a system to recover equilibrium, i.e., convergence of transient process (converge)



Basic requirements for control systems

1. Stability



Basic requirements for control systems

1. Stability ---(converge)

2. Accuracy : refers to size of a steady-state error when the transient process ends. (small)

   (Steady-state error = desired output – actual output)

3. Swiftness: refers to duration of transient process (quick)

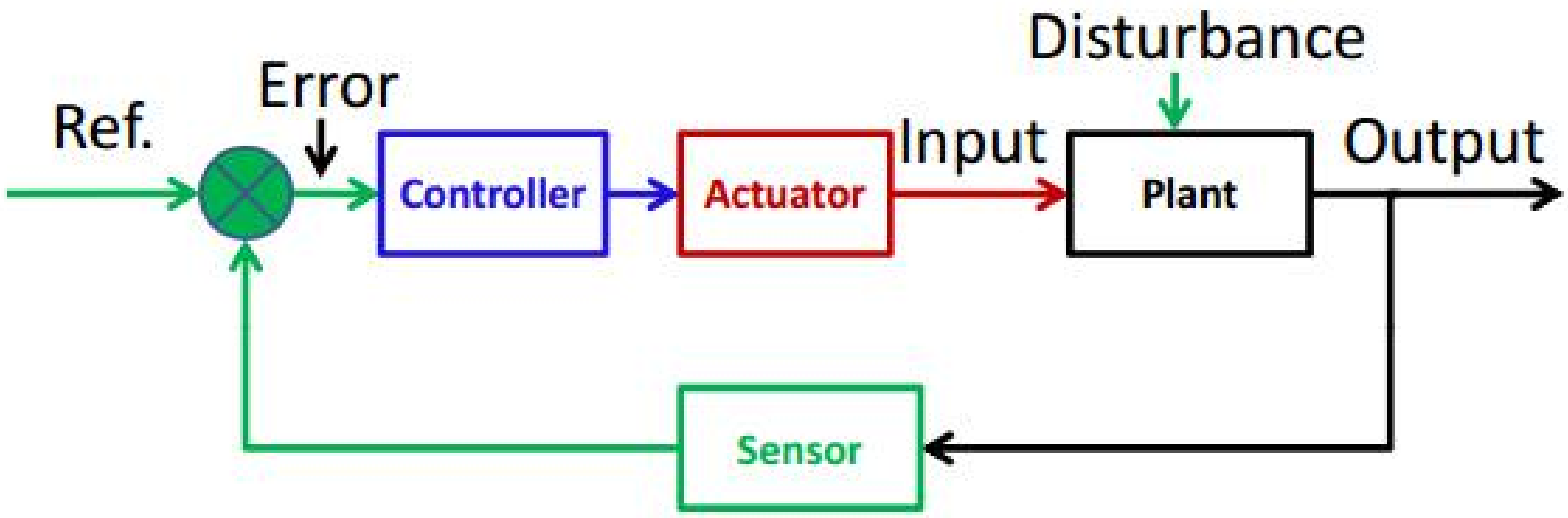
Basic requirements for control systems

Note

For a control system, the above three performance indices (stability, accuracy, swiftness) are sometimes contradictory.

In design of a practical control system, we always need to make compromise.

Basic requirements for control systems



- Regulation process: From initial equilibrium state to a new equilibrium.

Basic requirements for control systems

- The whole regulation process can be divided into two stages:

- Transient process in which the controlled variable goes from the initial state to the final state.

- steady-state process in which the system output stay at a new equilibrium state and maintain at a constant value.

Time response C(t) of control system

- Steady-state response Css(t)

  - system output behaves as approaches infinity.

$$c_{ss}(\mathrm{t}) = \lim_{t \to \infty} c(t)$$

Time response C(t) of control system

- Transient response Ct(t)

 - the controlled variable goes from the initial state to the final state

$$\lim_{t\to\infty} c_t(t) = 0$$

Time response of systems

- The variable used to measure the performance of the control system is the error, e(t), which is the difference between the input or reference value, r (t), and the controlled variable, c(t).

$$\min \quad e(t) = r(t) - c(t)$$

(Steady-state error = desired output – actual output)

Typical external signals

- In order to evaluate the performance of different system, it is necessary to select several test signals.
- Rules of selection

(1) Easy to generate

(2) Frequently encountered

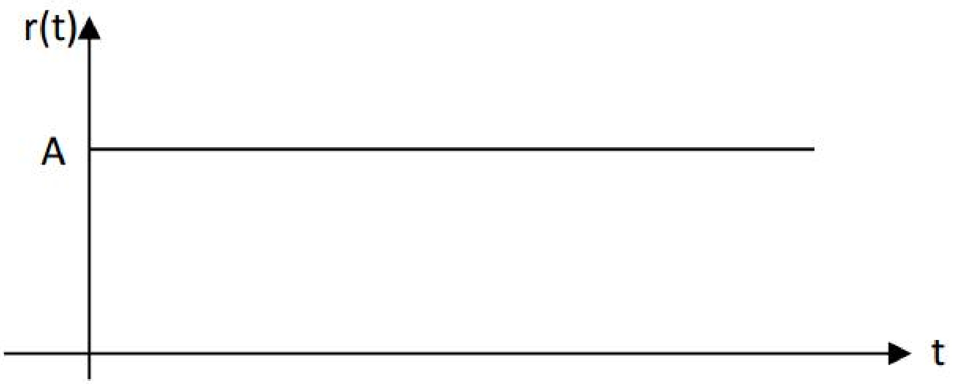
(3) Simple mathematical expression

Typical external signals

- Step function
- Ramp function
- Parabolic function
- Impulse function

Typical external signals

- Step-function (position signal)

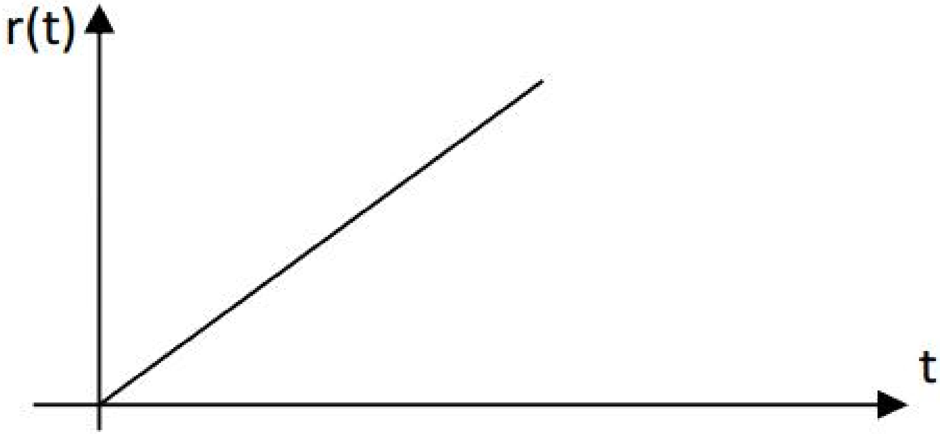


$$r(t)=\begin{cases} A, & t\geq 0 \\ 0, & t<0 \end{cases}$$ or $$r(t)=A\bullet 1(t)$$

e.g. if the input is the angular position of a mechanical shaft, the step input represents a sudden rotation of the shaft.

Typical external signals

- Ramp-function (velocity signal)

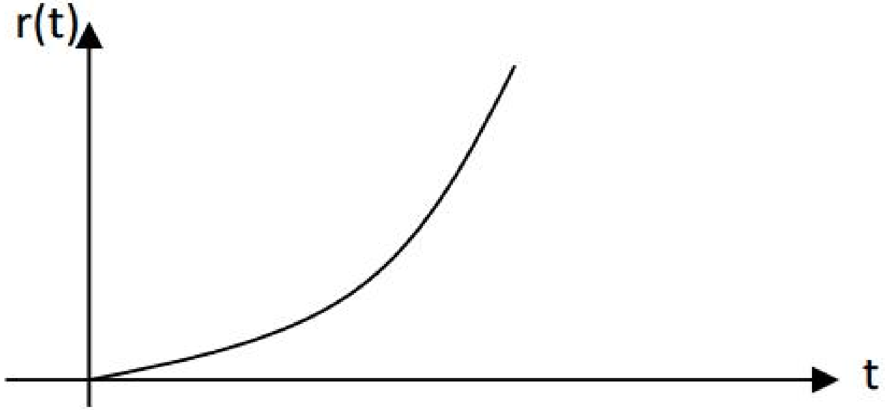


$$r(t)=\begin{cases} At, & t\geq 0 \\ 0, & t<0 \end{cases}$$

e.g. If the input variable is of the form of the angular displacement of a shaft, the ramp input represents the constant-speed rotation of the shaft.

Typical external signals

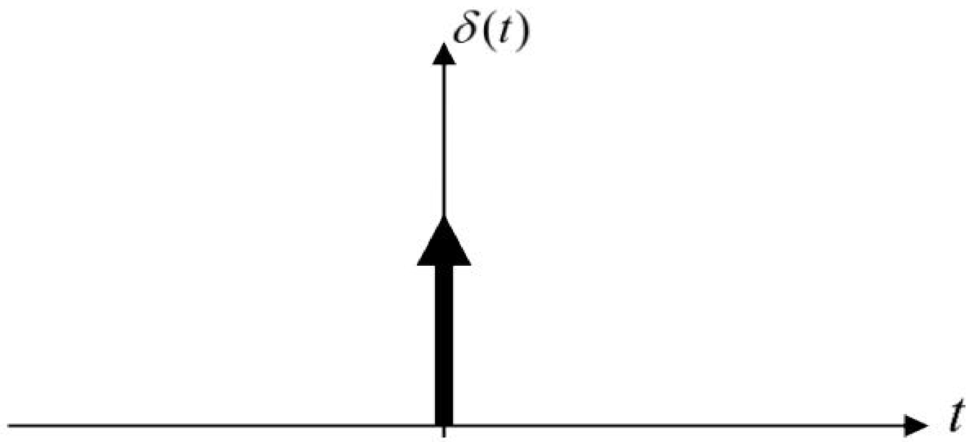
- Parabolic-function (acceleration signal)



$$r(\mathrm{t}) = \begin{cases} \mathrm{At}^2, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Typical external signals

- Impulse-function



$$r(t)=\begin{cases}\frac{1}{h}, & 0<t\le h\\ 0, & t<0 \ or \ t>h\end{cases} \quad \xrightarrow{h\to 0} \quad \delta(t)=\begin{cases}\infty, & t=0\\ 0, & t\ne 0\end{cases}$$

Note: The above impulse function does not exist in really, it is just mathematical definition.

Typical external signals

- All these test signals have the common features that they are simple to describe mathematically and easy to realize experimentally.
- Since step input function jumps from zero to a constant value, it is always chosen as a typical input signal to investigate system performance.

Basic requirements for control systems

- The system is expected to be affected Only by control input, NOT by disturbance.

- Two-fold tasks:

  (1) how the system responses to the external input?

  (2) how the system resists disturbances?

System specification in time-domain

Unit-step response

- Transient response

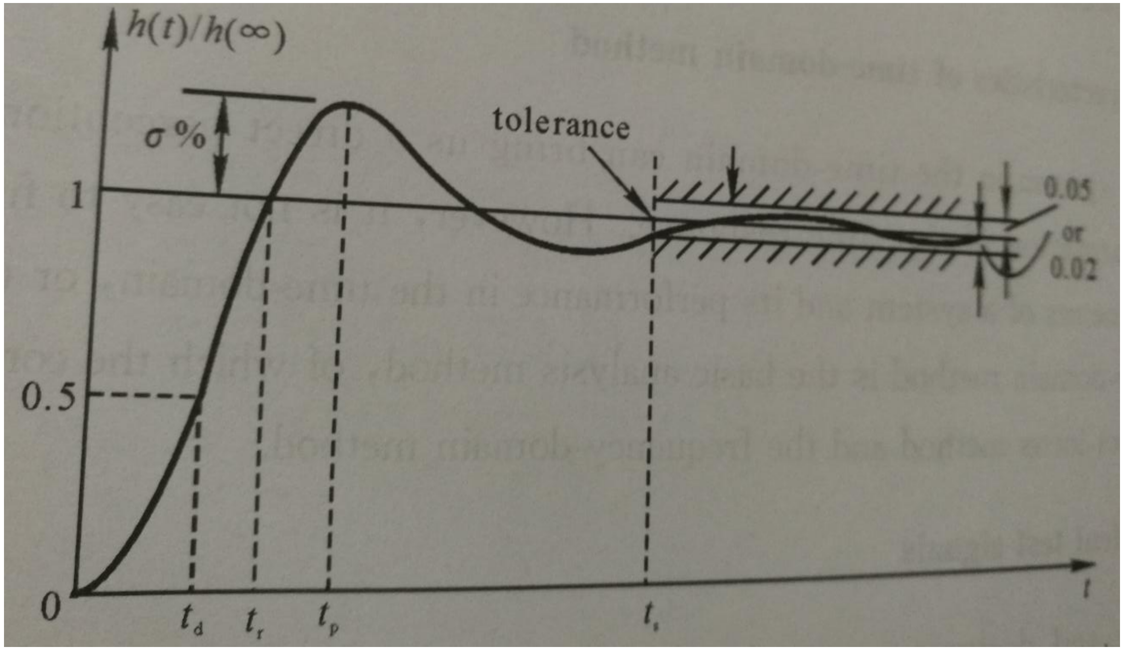
  -Dynamic characteristic

- -Steady-state response

  - steady-state characteristic

System specification in time-domain

Dynamic characteristic



Delay time t_d

Rise time t_r

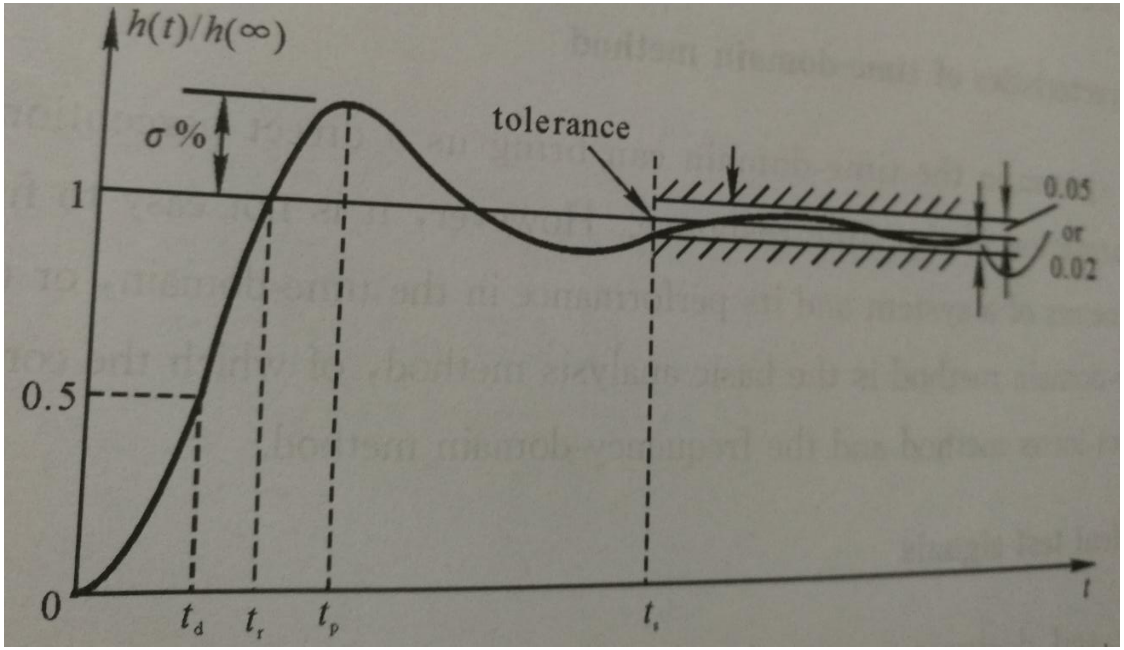
Peak time t_p

Settling time t_s

Maximum percent overshoot $\sigma\%$

System specification in time-domain

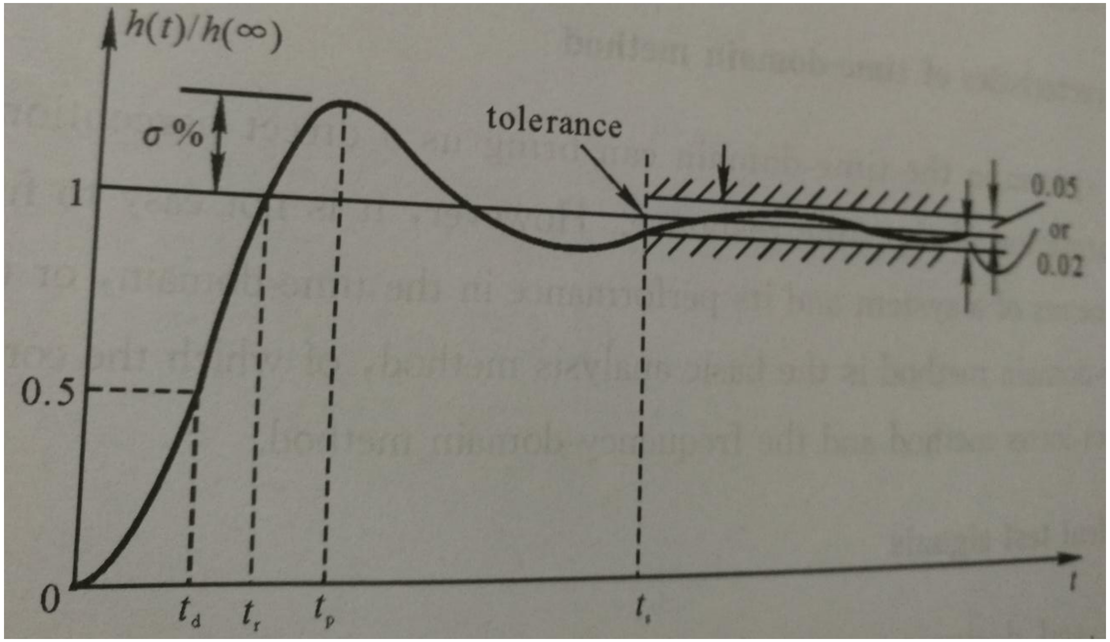
Dynamic characteristic



Delay time t_d : required for the response to reach half the final value the very first time.

System specification in time-domain

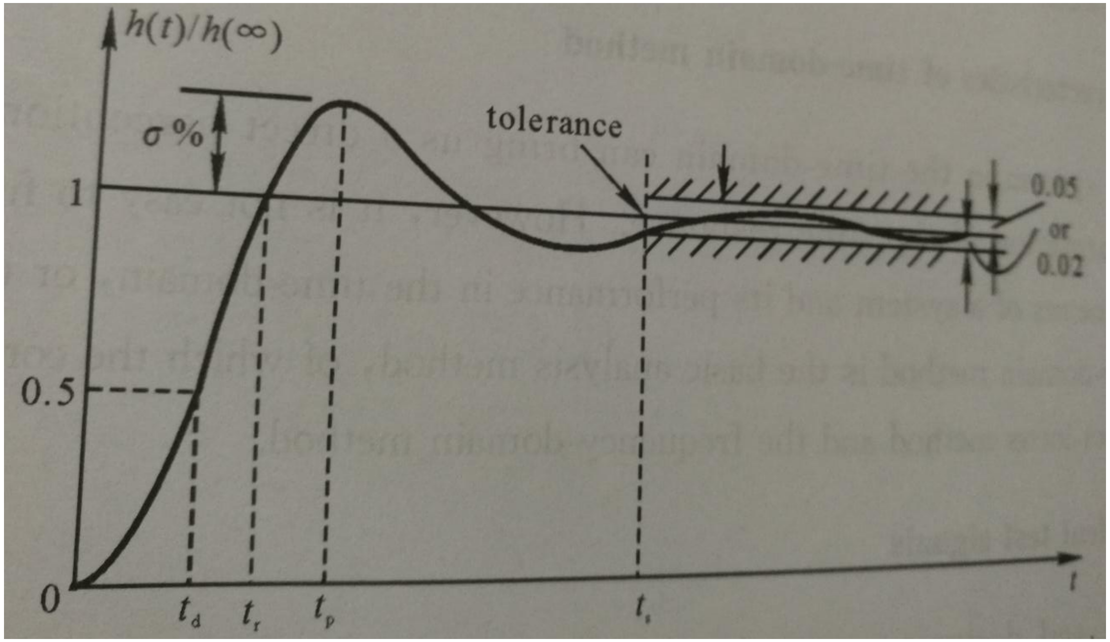
Dynamic characteristic



Rise time t_r: required for the step response to reach the final value first time, it can also be defined as the time required for the response to rise from 10% to 90%.

System specification in time-domain

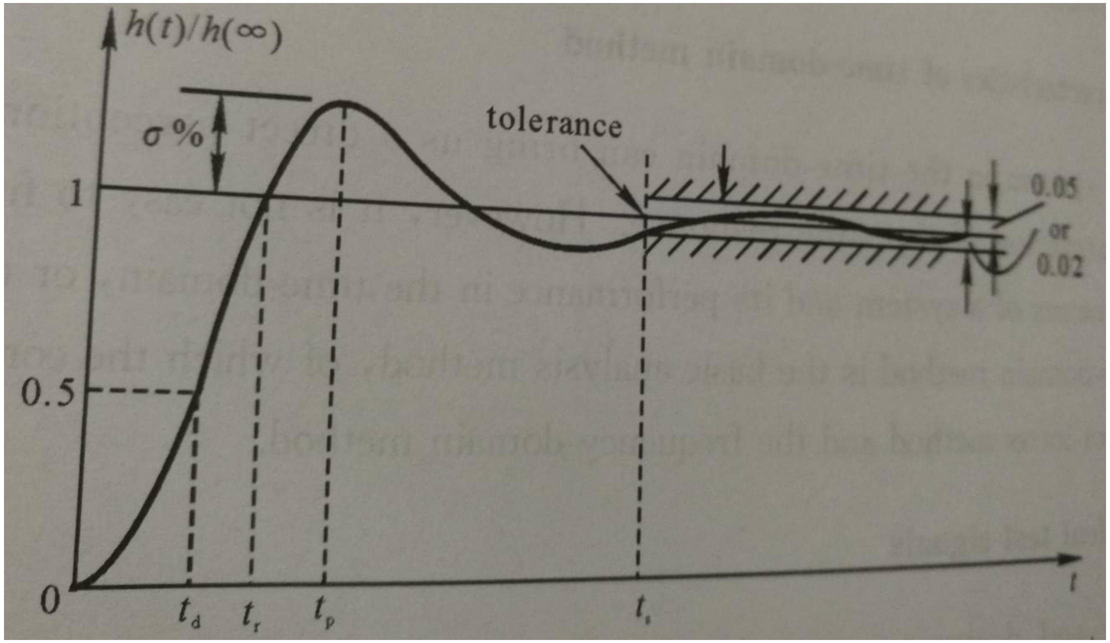
Dynamic characteristic



Peak time t_p: required for the response to reach the first peak of the overshoot.

System specification in time-domain

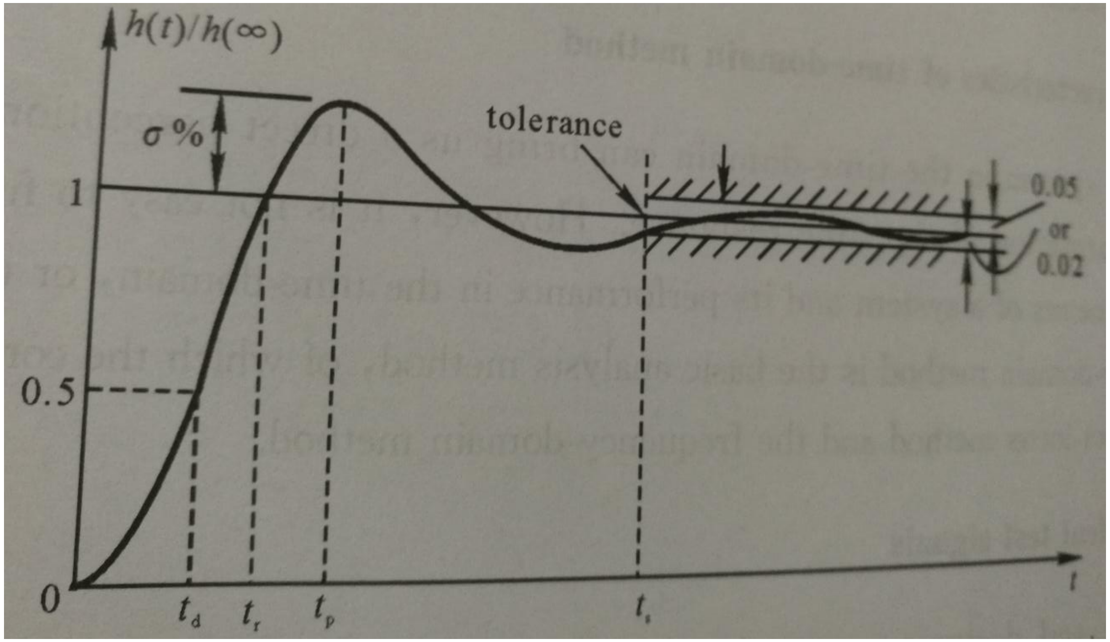
Dynamic characteristic



Settling time t_s : required for the response curve to reach and stay within a range about the final value of size specified by absolute percentage of the final value (2% or 5%)

System specification in time-domain

Dynamic characteristic



Maximum percent overshoot $\sigma\%$:

$$\sigma\% = \frac{h(t_p) - h(\infty)}{h(\infty)} \times 100\%$$

System specification in time-domain

Dynamic characteristic

The transient response of a control system may be described in terms of two factors:

1. The speed of response, as represented by the delay time, rise time, peak time, and settling time.
2. The closeness of the response to the desired response, as represented by maximum percent overshoot.

System specification in time-domain

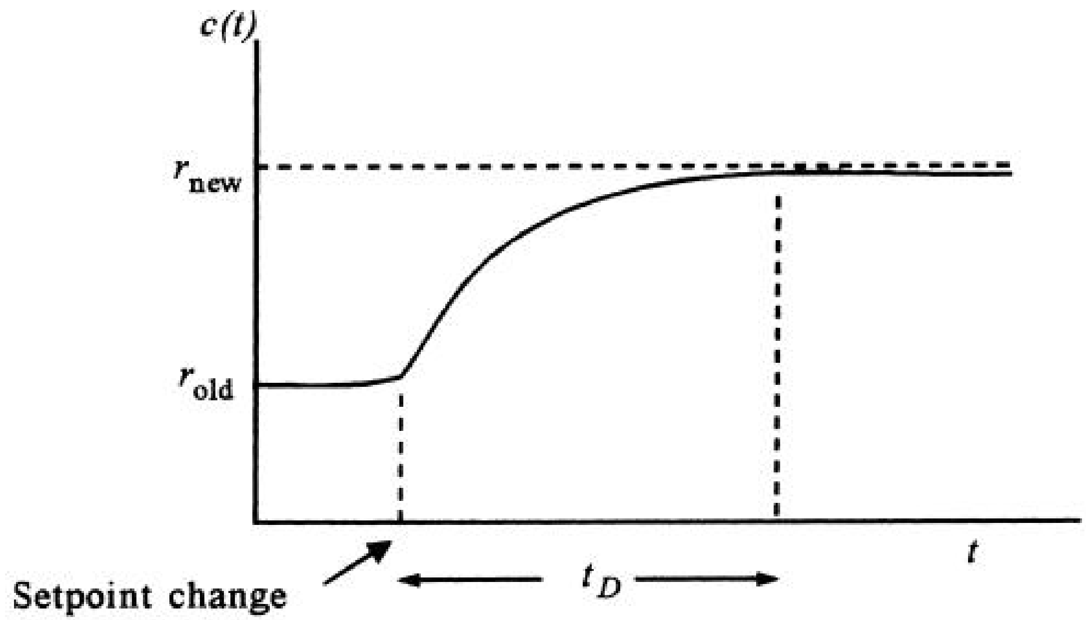
Steady-state characteristic

- The variable used to measure the performance of the control system is the error, e(t), which is the difference between the input or reference value, r (t), and the controlled variable, c(t).

Steady-state error = desired output – actual output

System Evaluation Criteria

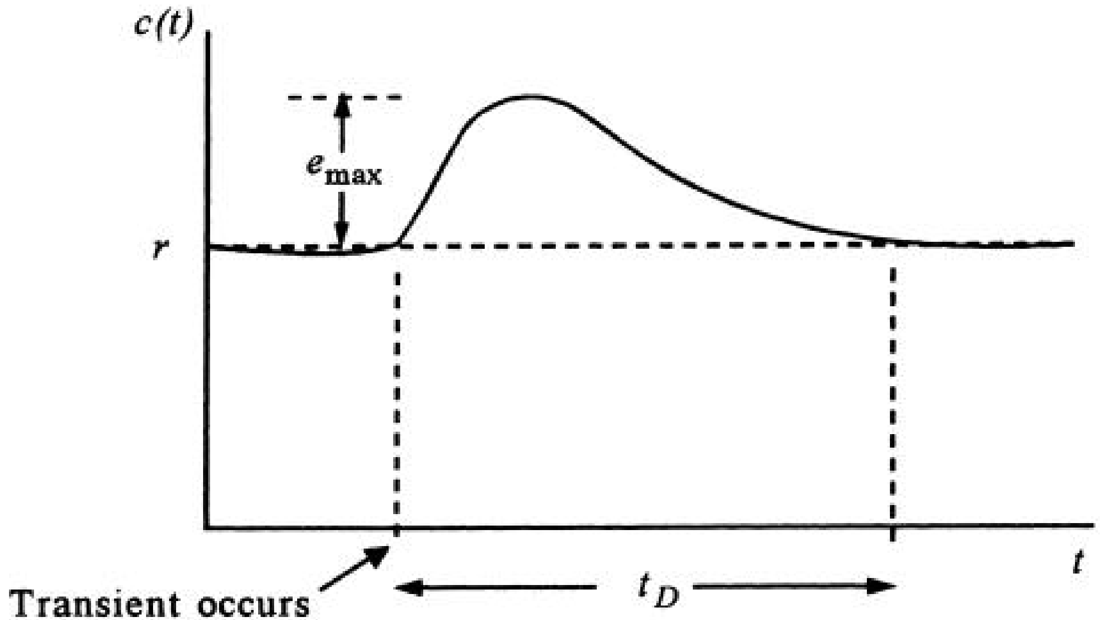
Damped response



One of the measures of control system performance is how the system responds to changes of setpoint or a transient disturbance.

System Evaluation Criteria

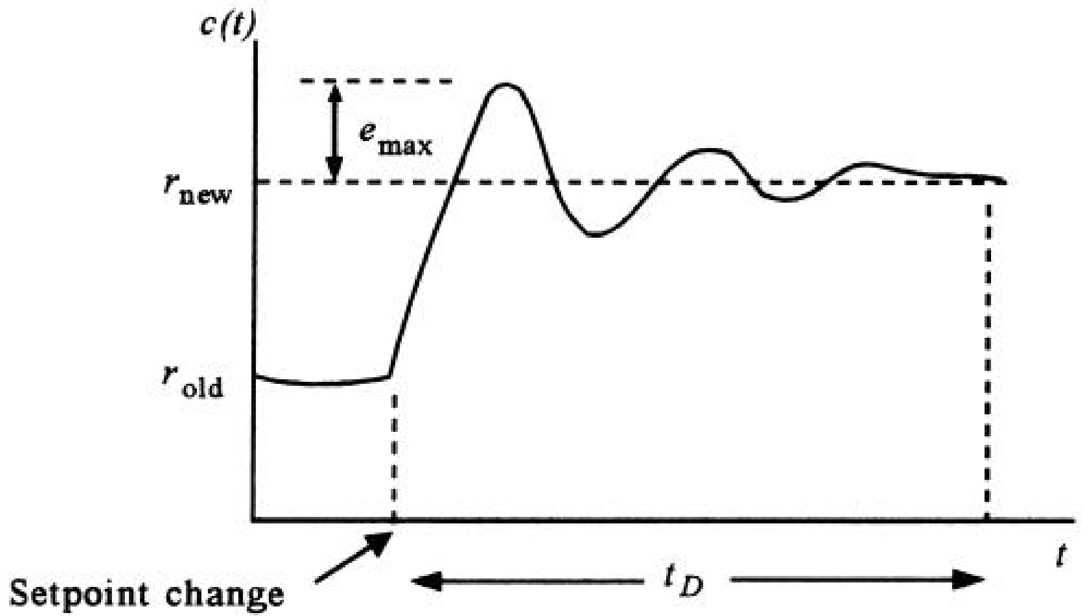
Damped response



One of the measures of control system performance is how the system responds to changes of setpoint or a transient disturbance.

System Evaluation Criteria

Cyclic response

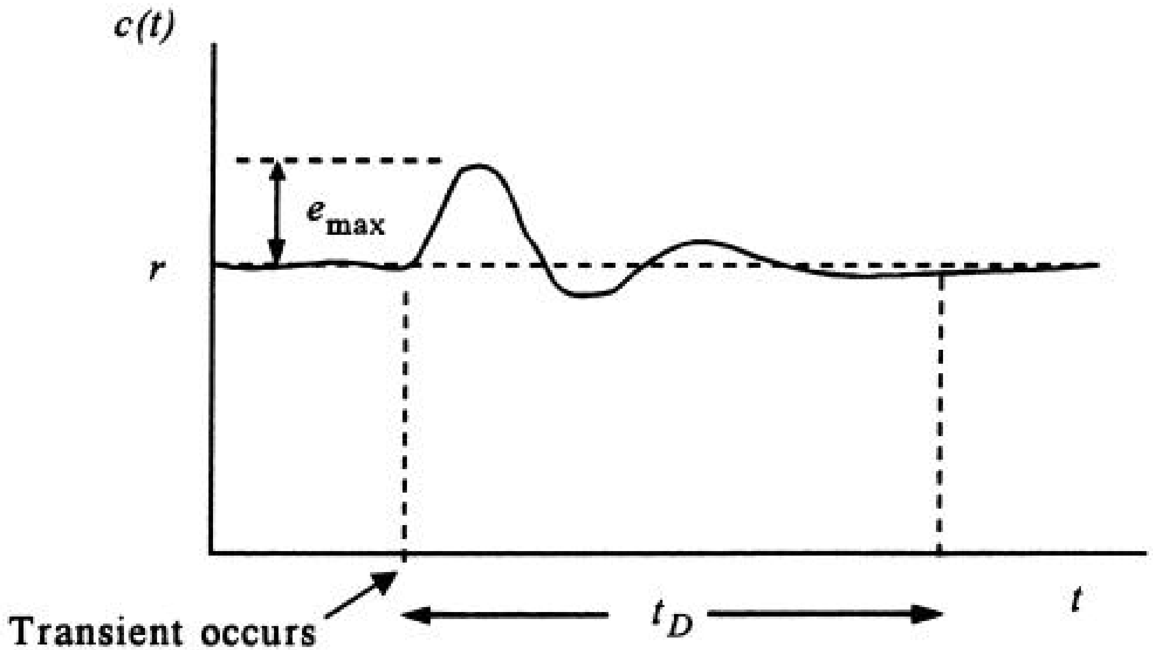


a) Setpoint change oscillations

In cyclic or underdamped response, the variable will exhibit oscillations about the reference value

System Evaluation Criteria

Cyclic response



b) Transient change oscillations

In cyclic or underdamped response, the variable will exhibit oscillations about the reference value

System Evaluation Criteria

Academic measures:

$$e(t)=r(t)-c(t)$$

- Integral Error (IE)
- Integral Squared Error (ISE)
- Integral Absolute Error (IAE)
- Integral Time-weighted Absolute Error (ITAE)

System Evaluation Criteria

Academic measures:

- Integral Error (IE)

$$IE = \int_0^\infty e(t)dt$$

- cannot penalize the undamped oscillation

System Evaluation Criteria

Academic measures:

- Integral Squared Error (ISE)

$$ISE = \int_0^\infty e^2(t)dt$$

- Often leads to fast responses, but with considerable, low amplitude, oscillation

System Evaluation Criteria

Academic measures:

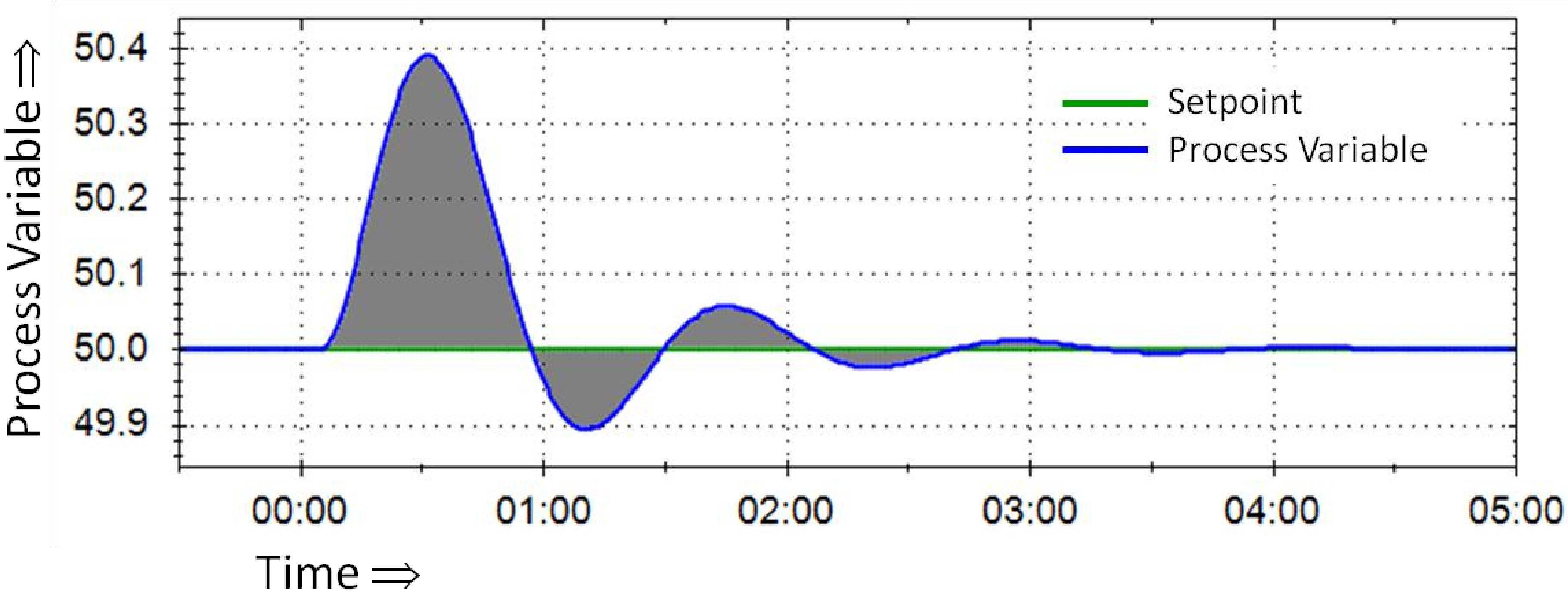
- Integral Absolute Error (IAE)

$$IAE = \int_0^{\infty} |e(t)|\, dt$$

- tends to produce slower response than ISE optimal systems, but usually with less sustained oscillation.

System Evaluation Criteria

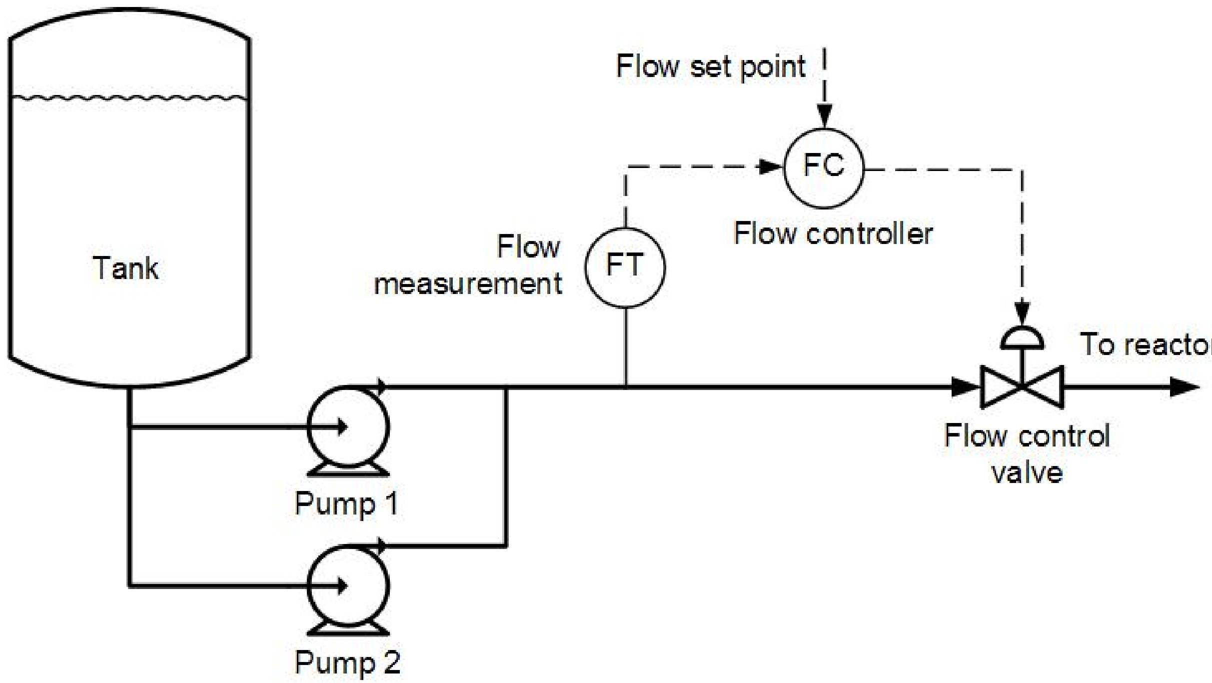
- Integral Absolute Error (IAE)



Shaded areas indicate the integral of absolute error after a disturbance to a process.

System Evaluation Criteria

- Integral Absolute Error (IAE)



The total flow rate will be disturbed when pump 2 starts up.

System Evaluation Criteria

Academic measures:

- Integral Time-weighted Absolute Error (ITAE)

$$ITAE = \int_0^\infty t\left|e(t)\right|dt$$

- settle much more quickly than the other three tuning methods.

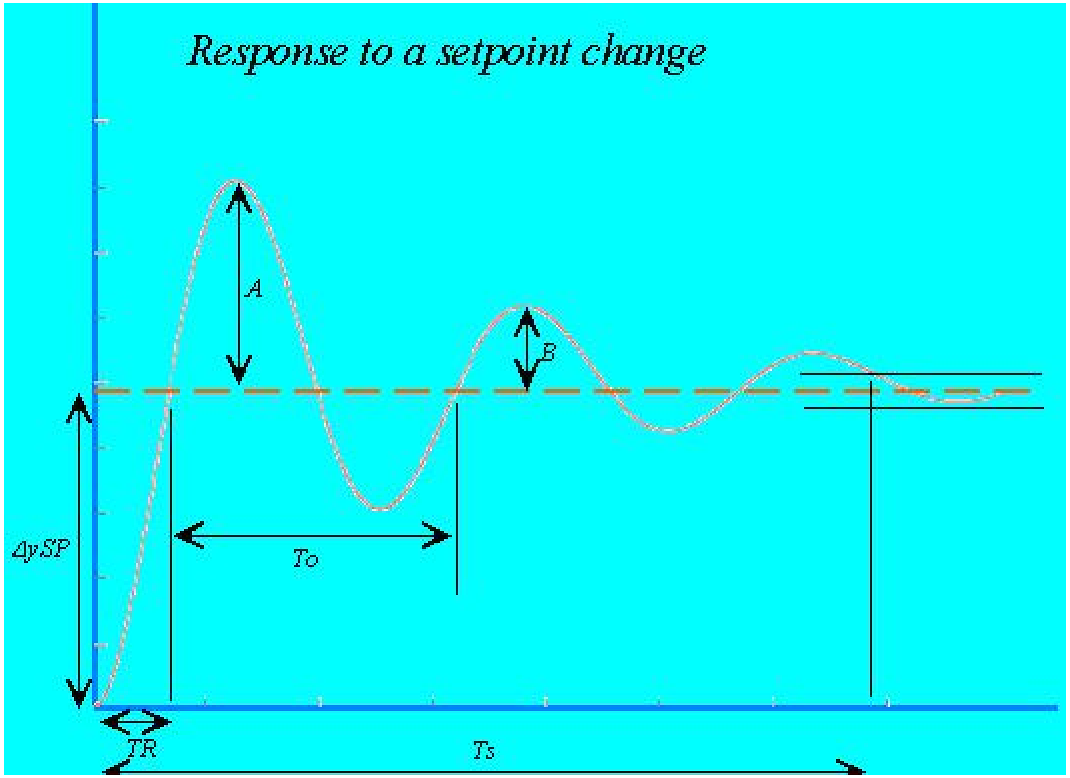
System Evaluation Criteria

Academic measures:

- ISE is greater than IAE to penalize the large error, while IAE is greater than ISE to eliminate the small error. And ITAE can penalize the error that exists in system for a long time.
- However, the above four measures can't be used in practical system comparisons is that they require a carefully controlled experiment, where only a particular parameter is changed in a predefined way.

System Evaluation Criteria

Practical measures:

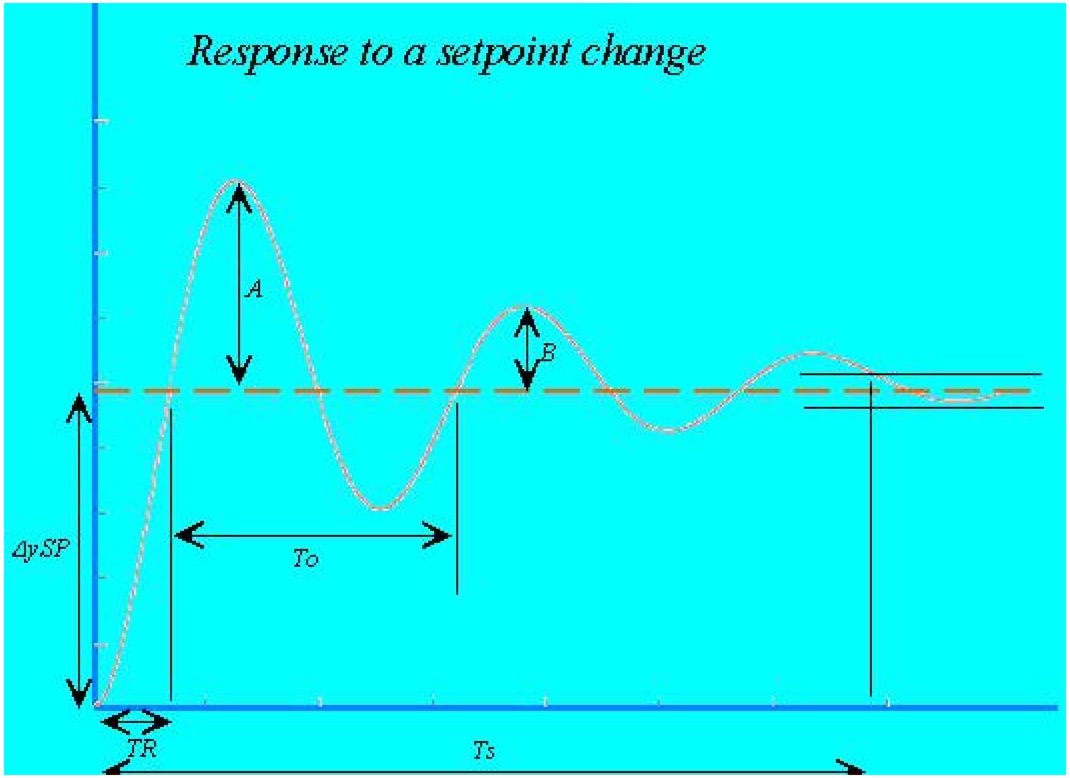


- Maximum absolute overshoot A:

The largest deviation during the regulation process

System Evaluation Criteria

Practical measures:

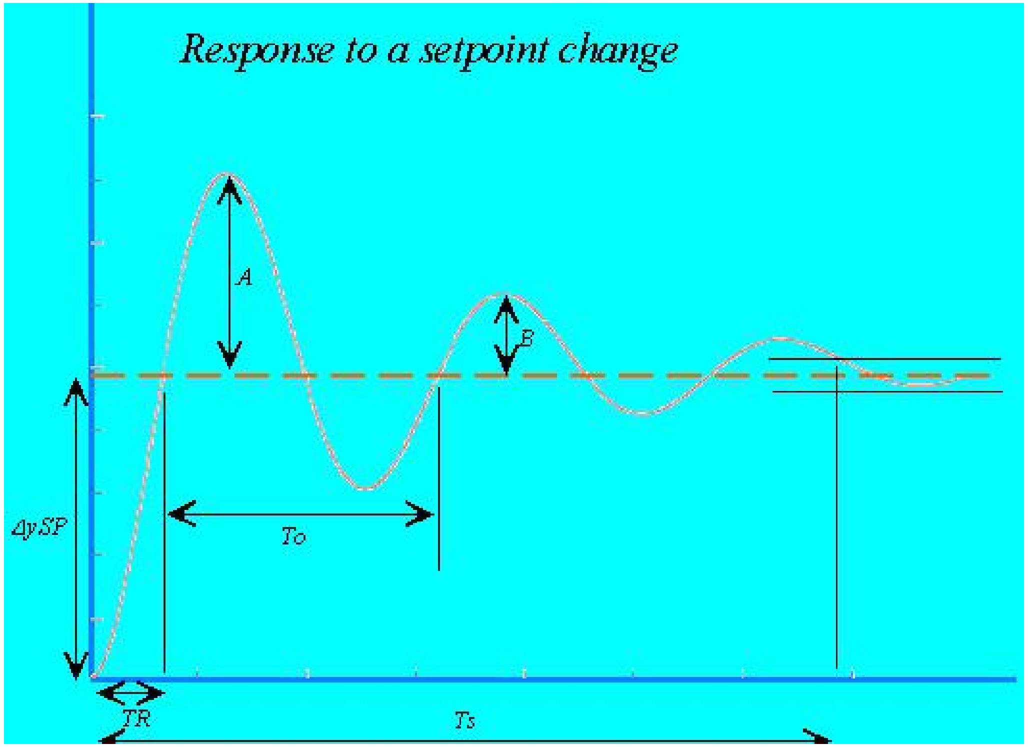


- Period of oscillation To:

the length of time for the oscillation to complete one cycle

System Evaluation Criteria

Practical measures:



- Decay ratio: A/B

the ratio of the height of successive peaks of the process response

System Evaluation Criteria

Practical measures:

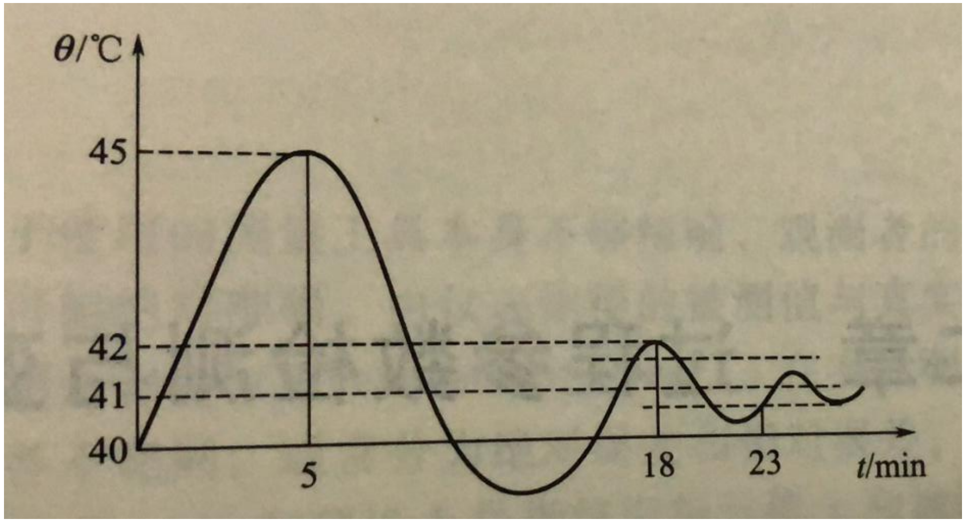
1. The most widely used measure of practical control performance is the decay ratio as it gives a good indication of the stability of the controlled response.
2. An often used rule for controller tuning is the controller is tuned to produce a decay ratio of 4/1 (the second peak is a quarter the height of the first).

Commonly used (4:1) ----(10:1)

System Evaluation Criteria

Exercise:

Figure 1 shows a regulation process curve of a temperature control system, please write the maximum absolute overshoot, maximum percent overshoot, decay ratio, period of oscillation steady-state error. If process requirement for temperature is (40 ± 2) $^{\circ}C$, do you think whether it satisfy the process requirement.



Outline

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2.2 Typical test signal

2.3 System specifications in time-domain

2.4 System evaluation criteria